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Lagrangian particle tracking and measurements of particle rotation elucidate dynamics of pulsatile particle-laden pipe flow

Lagrangesche Partikelverfolgung und Messung der Partikelrotation erlauben detaillierte Einblicke in die Dynamik pulsierender partikelbeladener Rohrströmungen

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Abstract

Particle suspensions in pulsatile pipe flow are ubiquitous in industrial processes and physiological systems. An experimental challenge is the simultaneous three-dimensional (3D) measurement of the particle and fluid motion, which is necessary to fully characterize these types of flows. In a long pipe experiment (up to $20\,\mathrm{m}$) we illuminate a pipe volume (having a diameter of $D = 28\,\mathrm{mm}$ and a length of about $4\,D$) with a pulsed laser and monitor it with four high-speed cameras to elucidate the particle-flow interaction. A hexagonal prism minimizes optical distortions, and tracers are added to the liquid to track them with the Shake-The-Box (STB) algorithm. In order to produce a suspension of about 1% particle volume fraction, we add hydrogel particles with a diameter of about $0.4\,D$. Hydrogel particles are transparent and therefore provide ideal optical access to the fluid flow even in their vicinity. In addition, the hydrogel interface can be detected by tracers adhering to its surface. This allows us to measure the hydrogel trajectory and even the rarely measured quantities of particle rotation axis and angular velocity. The experimental setup and the calibration and measurement procedure are explained in detail.

Zusammenfassung

Partikelsuspensionen in pulsierender Rohrströmung sind allgegenwärtig in Industrieprozessen und physiologischen Systemen. Eine experimentelle Herausforderung ist das simultane tomographische Messen der Partikel- und Fluidbewegung, welches für die vollständige Charakterisierung solcher Strömungen notwendig ist. In a einem langen Rohrexperiment (bis zu $20\,\mathrm{m}$) beleuchten wir einen Rohrvolumen (mit Durchmesser $D = 28\,\mathrm{mm}$ und einer Länge von ungefähr $4\,D$) mit einem pulsierenden Laser und beobachten dieses über vier Hochgeschwindigkeitskameras um die Partikel-Strömungs-Interaktionen zu erfassen. Ein hexagonales Prisma miniminiert die optischen Verzerrungen und Tracerpartikel innerhalb der Strömung werden über den Shake-The-Box (STB) Algorithmus verfolgt. Um eine Suspension mit einem Partikelvolumenanteil von etwa 1% zu erzeugen, fügen wir Hydrogelpartikel mit einem

Durchmesser von etwa $0{,}4\,D$ hinzu. Hydrogelpartikel sind transparent und bieten daher einen idealen optischen Zugang zur Flüssigkeitsströmung auch in direkter Partikelnähe. Darüber hinaus kann die Hydrogel-Grenzfläche durch Tracer, welche an ihrer Oberfläche haften, erkannt werden. Hierdurch lassen sich die Trajektorie des Hydrogels und sogar die selten gemessenen Größen Rotationsachse und Winkelgeschwindigkeit der Partikel messen. Der Versuchsaufbau sowie das Kalibrierungs- und Messverfahren werden im Detail erläutert.

Introduction

Particle-laden flows are ubiquitous in engineering and nature, reaching from the flow of slurries, sedimentation processes to the cardiovascular blood flow in living organisms. Most of these flows are time dependent and e.g. blood flow features in addition a pulsatile mass flux. Currently little is known, how particles move in such a flow environment. To better understand the dynamics of such flows, we investigate the four-way coupling between large inertial particles, a fluid flow and the wall geometry in a pulsatile pipe flow experiment. A pulsatile flow is described by the Reynolds number $Re = U\,D/\nu$ (with the mean flow velocity U, the pipe diameter D and the kinematic viscosity ν), the Womersley numbers $Wo = 0.5\,D\sqrt{2\pi\,f/\nu}$ as the nondimensionalized pulsation frequency f and the amplitude $A = \widehat{U}/U$ (as the ratio between peak velocity $\widehat{U}$ and mean velocity U). The main parameters of a (spherical) particle suspension are the particle diameter D_p, their volumetric fraction in the fluid flow ϕ and their density, which is matched to the liquid in our experiments.

A challenge for the measurement techniques is the simultaneous observation of the fluid flow and the particle motion to resolve their interactions. Modern optical methods like Tomographic Particle Imaging Velocimetry (PIV) (Scarano 2013) or Lagrangian Particle Tracking (LPT) with the Shake-The-Box (STB) algorithm (Schanz et al. 2016) allow high-resolution measurements of all three velocity components (3C) within a volume (3D). However, few of these methods have been applied to multiphase flows, because here the optical accessibility is limited due to the existence of different materials with varying refractive indices. A recent approach for investigating these flows is the usage of transparent superabsorbent polymers or hydrogel particles. These materials swell up in water leading to a large mass fraction of aqueous solutions within the particles. Therefore, the hydrogel particles have a very similar density (and are hence neutrally buoyant) and, most importantly, match the refractive index of the water-based working fluid. This enables PIV measurements at high concentrations of hydrogel particles in the flow (up to $\phi = 0.184$ (Zhang and Rival 2018)). The hydrogel can also be imbued with fluorescent dye to visualize them and calculate the particle trajectories. From the same camera recordings the fluid flow can be obtained with 2D PIV (Zade et al. 2018). In a study by Klein et al. (2013) the hydrogel particles were specifically marked with tracers on their interface, which allowed it to determine the rotation of the particles in a turbulent von Karman swirling flow configuration using 3D3C LPT. In pipe flow the particle rotation is anticipated to be relevant for the inertial particle migration (Zhang et al. 2016) and the onset of turbulence in particle-laden flows (Yu et al. 2013), but so far it has not been measured. Additional challenges stem from the refraction change at the curved pipe wall.

In this work, we will simultaneously determine the flow field, the particle trajectories and rotation. Klein et al. (2013) used an in-house LPT algorithm in combination with three cameras and tracked about 200 tracers per time step. In our setup we use four cameras, which substantially improves the verification of the tracer positions and reduces the risk of false tracks (so-called ghosts). The powerful Shake-The-Box algorithm (Schanz et al. 2016) allows substantially higher tracer densities compared to other LPT methods and therefore a better spatial resolution (we typically obtain 3000 trajectories) . Such a resolution is essential for the reconstruction of the Eulerian velocity field from LPT data (Jeon et al. 2022). In order to obtain the hydrogel propagation and rotation, we exploit the effect that the tracers naturally adhere to the hydrogel interface, instead of specifically grafting the tracers. This is less laborious and results in dense surface markers, which simplifies the processes of identifying the hydrogel particles and tracking their motion. In this manuscript, we first present the experimental setup and then explain the methods for recording and processing the STB measurements to determine both the fluid velocities and particle motion. In the last section, we show first results of applying this technique to a pulsatile particle-laden pipe flow.

Measurement setup

The Shake-The-Box measurements are performed in a long pipe experiment (length up to $20\ \mathrm{m}$, diameter $D = 28\ \mathrm{mm}$, see Fig. 1). The working fluid is a water-glycerol ($10\ \mathrm{wt\%}$) mixture laden with large hydrogel spheres ($D_p \cong 11.5\ \mathrm{mm} = 0.41\ D$) with a volume concentration of up to 1%. The pipe consists of individual borosilicate glass segments mounted to an aluminum strut structure. The flow is created by pulling fluid with a large hydraulic piston driven by a linear actuator with a servo motor. By oscillating the servo motor speed, a pulsatile flow with adjustable frequency, amplitude and waveform can be generated. An open reservoir for seeding the large particles and tracers at the entrance connects to the pipe via a trumpet shaped entrance nozzle. At the end of the pipe, the large particles are captured in a second reservoir that connects via a filter to the driving system.

For multiphase flow measurements, large transparent hydrogel particles are seeded into the flow. These are initially small $1.9\ \mathrm{mm}$ polyacrylate particles that swell up in water to their final size. Due to the polymer part, the swollen hydrogel particles are slightly heavier than water ($\sim 1.01\ \mathrm{g/cm^3}$). To minimize the density mismatch, we use a water-glycerol mixture ($1.027\ \mathrm{g/cm^3}$) as working fluid. Initially the hydrogel particles are slightly lighter, but over several hours the hydrogel absorbs the water-glycerol and slowly increases in density. For an intermediate period of about three hours, the particles can be regarded as neutrally buoyant, even for small inertial flow velocities and despite their large diameter. The buoyancy can be roughly checked by driving the flow at $Re \cong 300$ and observing, if the particles are properly lifted by inertial forces or roll along the (upper or lower) pipe wall.

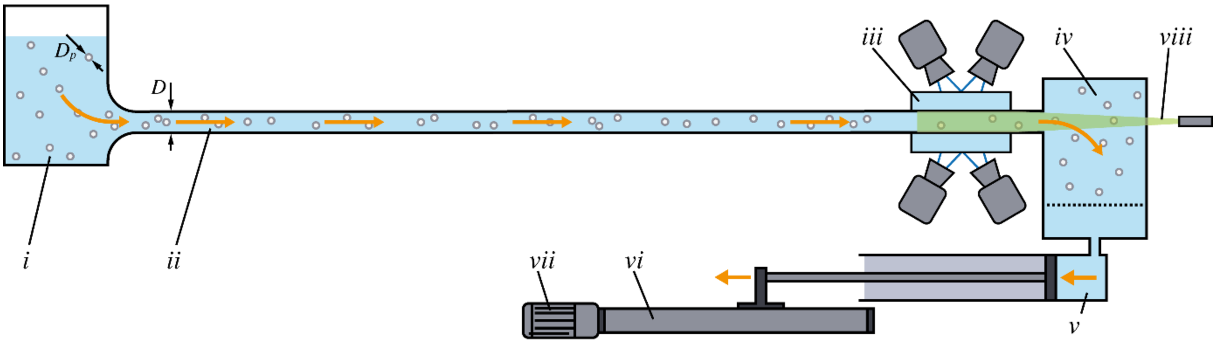

Fig. 1: Sketch of the pipe flow experiment. The suspension flows from an inlet tank (i) into the glass pipe (ii), passes a STB-measurement volume (iii) and ends in an outlet tank (iv). The large particles are collected in this tank, while the fluid is pulled through a mesh into the piston (v), driven by linear actuator (vi) with a servo motor (vii). A laser illumination (viii) is provided axially through the outlet tank.

The measurement equipment for STB (displayed in Fig. 2) is located at the end of the pipe. The tomographic setup consists of four high speed cameras, a pulsed laser illumination, a refractive index matching prism and the processing software Davis 10 from LaVision.

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(a) (b)

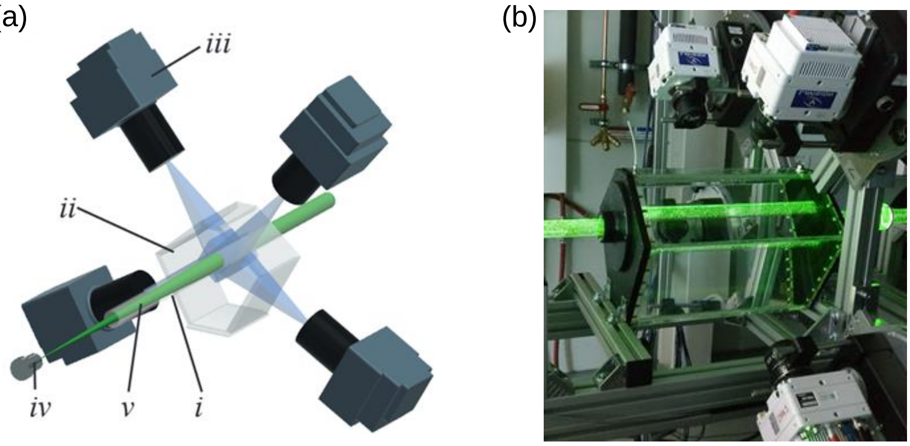

Fig. 2: Measurement setup for Lagrangian Particle Tracking with the Shake-The-Box algorithm. In the sketch in (a) the placing of a pipe section (i) inside a hexagonal refractive index matching prism (ii) can be seen. Four high speed cameras are in an in-line arrangement (iii) with the laser illumination being perpendicular to all cameras. In the photo in (b) the axial laser illumination of the pipe is clearly visible.

For Lagrangian Particle Tracking the signal of the scattered light of the tracers has to be of better quality than e.g. for an PIV analysis, because here the individual tracers are identified and tracked over time. We achieve Gaussian-distributed light intensity signals by using polyamide-12 spheres with mean diameter of $60\,\mu\mathrm{m}$ (type: Orgasol 2002 ES6 NAT3 from Arkema) as tracer particles. These tracers have a nominal density of $1030\,\mathrm{kg/m^3}$, which matches the water-gylcerin mixture very closely.

To minimize the optical distortion of the camera images stemming from the refraction at the round pipe wall, the pipe segment used for the volumetric measurements is placed inside a hexagonal refractive index matching prism made from acrylic glass. The prism is filled with the working fluid to match the refractive index of borosilicate glass pipe wall ($n = 1.473$). In our setup, the choice of the working fluid is limited due to the density matching with the large particles. Consequently, the water-glycerol ($n = 1.344$, calculated based on Miessner et al. (2007)) does not fully match the glass, resulting in some remaining distortions. These can mostly be compensated with the calibration procedure.

For the image acquisition, four high speed cameras type VEO 640L from Phantom ($1.4\,\mathrm{kHz}$ @ $2560 \times 1600\,\mathrm{Px}$) are mounted in a radial plane around the prism. Each camera faces the measurement volume through one the surfaces of the prism. The angle between the individual camera views is $80°$ (see Fig. 2 (a)). The axial length of investigated pipe section ($4\,D - 10\,D$) and spatial resolution of the cameras ($9 - 27\,\mathrm{px/mm}$) can be set, by adjusting the distance to the prism.

The tracers in the measurement volume are illuminated with a pulsed dual-cavity $517\,\mathrm{nm}$ Nd:YAG laser ($2 \times 50\,\mathrm{mj}$ @ $1\,\mathrm{kHz}$, DM50-527-DH from Photonics) positioned at the end of the pipe. The expanded beam passes through the outlet tank into the pipe to illuminate the full pipe cross-section in the measurement volume. Compared to a radial illumination, this configuration avoids remaining reflections from the pipe wall. Additionally, with the radial in-line arrangement of the cameras, each camera receives light from the same $90°$ scattering angle, resulting in uniform tracer intensities. Consistent intensities are especially beneficial for the STB-algorithm tracking.

Image Pre-Processing and Calibration

A pre-processing of the raw data images is applied to enhance the contrast and visibility of the tracers. An example of a raw data image and a pre-processed one, is shown in Fig. 3 (b) and (c). We apply a time-averaged filter (of at least 7 frames) to remove stationary signals stemming e.g. from remaining reflections and tracers sticking to the pipe wall. To achieve a similar signal-to-noise ratio over the entire field of view (despite the Gaussian intensity profile of the laser intensity) a spatial moving average (over $8\,\mathrm{px}$) is used to smooth out the variations in space. As an optional last step, a Gaussian-blur smoothing, and sharpening filters can be used to increase the visibility of the tracers by reducing any remaining noise and increasing the signal of the tracers.

A volumetric calibration maps the tracer positions on the camera sensors into the three-dimensional space of the measurement volume. In the first calibration step, a volumetric calibration target with an even grid dot-pattern is inserted into the pipe (see Fig. 3 (a)). The plate is tailor-made to maximally fill out the measurement volume. By recording the plate with each camera, the predefined dot pattern can be recognized in the Davis 10 software, which calculates a perspective calibration (using a third order polynomial camera model). This calibration needs to be further refined to allow for tracer triangulation.

(a) (b) (c)

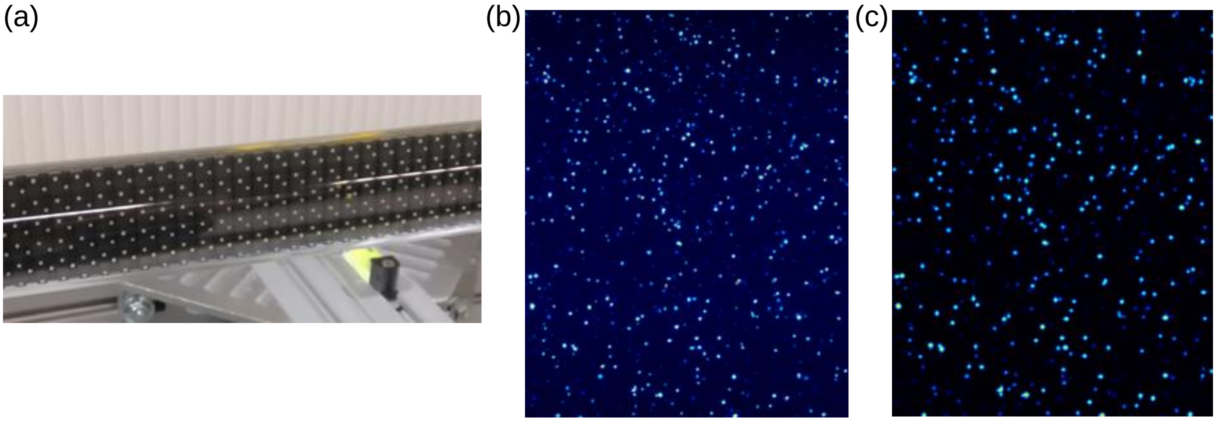

Fig. 3: (a) Custom double-sided volume calibration plate with two layered even grid dot patterns. Example snapshot with a section of a tracer image before (b) and after (c) image pre-processing.

In the second step of the camera calibration, the tracers within the fluid are illuminated with the laser and the cameras get focused so that all tracers in the volume appear sharp. The Davis 10 contains a powerful volume self-calibration algorithm (Wieneke 2008), which uses a set of recorded flow images (with the tracers in motion) to reduce the remaining disparity of the initial perspective calibration. In our procedure, the recorded pipe section is split into up to $10 \times 20 \times 20$ rectangular sub-volumes. In each volume, the residual error when triangulating tracers from the four camera views is mapped. Based on this disparity map, the calibration for each camera is refined. By repeating this procedure (typically about 4-6 iterations) the averaged remaining disparity reduces to $0.25\,\mathrm{px}$, which is well suited for volumetric measurements. Due to the remaining refractive index differences in the experiment, there are some optical distortions close to the wall, which cannot be compensated with the polynomial camera model. This leads to larger remaining disparities of up to $2\,\mathrm{px}$ near the walls. Based on the final iteration of the disparity field an optical transfer function is calculated which maps how tracers are projected in each camera for each sub-volume, this also includes unevenness in shape or size of the tracers due to astigmatisms or uneven illumination (Schanz et al. 2013).

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Processing and Post-Processing

The pre-processed images are analyzed with the Shake-The-Box algorithm (Schanz et al. 2016) implementation in LaVision's Davis 10 to calculate the particle trajectories. For the tracer detection a brightness threshold of 200 counts and allowed triangulation error of up to $3\ \mathrm{px}$ is chosen. Due to high efficiency of Shake-The-Box the computation of a single timestep takes only $2-3$ seconds (on a machine with an Intel Xeon E5-1650 6 core processor and 32 GB of RAM). On average, approximately 5000 tracers are triangulated, and 3000 tracks are identified per timestep. The average track length is 15 timesteps. Some remaining ghost tracks can be removed using a spatial median filter applied to the trajectory field, and by masking out any false tracks outside the pipe volume.

To convert the Lagrangian Trajectories to an Eulerian grid velocity field, the Vortex-in-Cell-sharp (VIC#) method (Jeon et al. 2022) (as an updated version of the Vortex-in-Cell-plus method by (Schneiders and Scarano 2016)) is applied. For our experiment, the velocity field is reconstructed with a $1\ \mathrm{mm}$ $(0.036\ D)$ grid size. Since the Davis 10 implementation of VIC# does not support wall boundaries in the measurement volume, the flow field is extrapolated into the volume beyond the domain of the pipe. A geometric mask is applied in post-processing to remove these areas form the velocity field. Compared to the STB processing, this reconstruction takes much more computational power with processing durations of about 60 seconds per timestep.

Tomographic measurements of particle-laden pipe flow

The goal of our setup is to simultaneously measure the fluid and particle motion time-resolved in a volume (3D3C). Since large particle typically yield strong light reflections and block out the laser illumination, we use transparent hydrogel particles. Fig. 4 (a) displays a tracer image with a hydrogel particle. Since the material itself has almost the same refractive index as the working fluid, it does not cast a shadow and even tracers behind the particle can be captured.

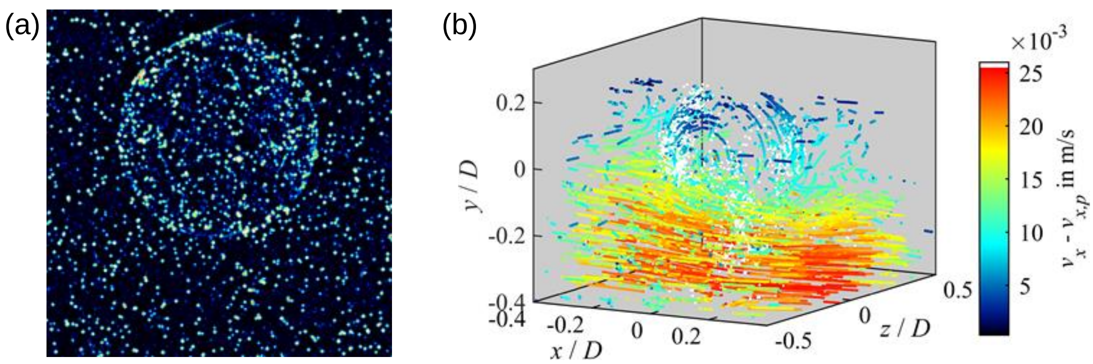

Fig. 4: (a) Raw data image of a hydrogel particle with tracers adhering to its surface. (b) Raw data of the corresponding Lagrangian tracer trajectories illustrate the motion of the hydrogel and the surrounding flow in the pipe.

An advantage of the hydrogel is that it does not require additional specific operations for image pre-processing or Shake-The-Box processing. If a successful volume-calibration was calculated, the same steps as for a single-phase fluid measurement can be used for particle-laden flow. In Fig. 4 (b) the trajectory field surrounding a hydrogel particle is shown. The particle itself appears as an empty sphere within the trajectory field with the surface markers displaying the exact position of the interface. Due to remaining effects of refraction a few falsely identified ghost particle at the interface and in the surrounding fluid remain. These usually do not match the true trajectories in magnitude or directions and can therefore be easily removed with a spatial median filter or velocity thresholding.

Extracting the particle position and rotation

After calculating the STB trajectory field, we can search for the particle and calculate its trajectory and rotation. In the method presented by Klein et al. (2013), 6 tracers were attached to hydrogel particles. We typically retrieve 200-1000 tracer tracks, which substantially simplifies the particle surface identification. To do so, the three-dimensional positions of all identified tracks for a timestep are mapped. Since we already know the approximate radius of the hydrogel, we can search the entire measurement volume for areas with a high concentration of tracks on a spherical shell and almost no tracks inside that sphere. The center of such an area is then defined as the center of mass of the hydrogel particle. After the position has been determined for every timestep, a moving average smoothing is applied to the hydrogel trajectory, to remove small deviations stemming from inhomogeneous track concentrations based on the hydrogel orientation or uneven illumination within the measurement volume.

After the hydrogel trajectory is determined, the Lagrangian data can be separated into trajectories belonging to the particle interface and the surrounding fluid, via a sphere-shaped mask around the particle`s center of mass. For the VIC# flow field reconstruction, only the tracks corresponding to the fluid flow are used, as the particle interface motion cannot be described by the Navier-Stokes equation and will lead to errors in the velocity field. From the hydrogel interface tracks we can not only determine the translatory motion of the particle but also the rotation speed and axis. This is done by subtracting the mean particle velocity from the interface track velocities. An example of this is displayed in Fig. 5 for the rotation of a large particle in a laminar steady flow. Following the shape of the tracks through multiple time steps reveals the direction of rotation. For the instantaneous rotation rate the rotation matrix between two consecutive timesteps is calculated for all tracked surface markers in both steps, using a least square quaternion-based method by (Horn 1987). Assuming that the rotation rate does not drastically change between two consecutive timesteps, a moving average filter is used to smooth the rotation rate. In the displayed example of a steady flow, the particle rotates around the z-Axis with a rotation speed of $1.45\ 1/s$.

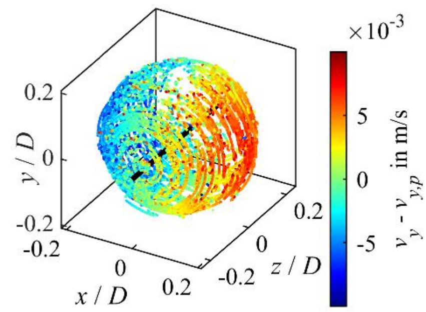

Fig. 5: Particle interface trajectories with velocity relative to the particles center of mass accumulated over 100 timesteps (corresponding to $0.5\ s$) for the motion of a hydrogel particle in a steady pipe flow at $Re \cong 600$. The surface tracks reveal a rotation around the z-Axis with $1.45\ 1/s$.

Particle motion in pulsatile particle-laden flow

An example of a STB measurement of a pulsatile particle-laden flow is shown in Fig. 6 for $Re = 1277$ and $Wo = 8$. In the decelerating fluid flow a large turbulent vortex structure appears in the wake of the particle. In this representation the tracer trajectories of the fluid flow are plotted together with the isosurfaces of the streamwise vorticity and the position of the hydrogel. Not shown here, for the sake of clarity, is the rotation of the hydrogel particle. The underlying flow instability has been previously observed in single-phase pipe flow after applying a geometric perturbation (Xu et al. 2020). In the future this method will allow it to elucidate how particles are able to trigger the same flow instability.

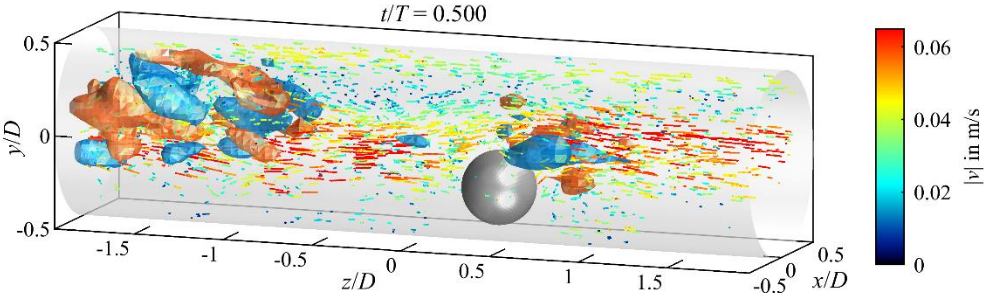

Fig. 6: Snapshot of the flow field around a hydrogel particle (grey sphere) obtained from Shake-The-Box measurements ($Re = 1277, Wo = 8$). The colored lines represent the individual fluid trajectories over $42\ \mathrm{ms}$ (21 timesteps). The red/blue surfaces correspond to the streamwise vorticity with $\omega_x = \pm 3.2\ U/D$. The flow direction is from left to right. A flow instability with large vortices arises in the wake of the hydrogel particle during the deceleration phase of the pulsatile pipe flow.

Conclusion

We performed Shake-The-Box measurements of particle-laden pipe flow yielding the 3D3C fluid velocity field in addition to the particle trajectory and rotation. By using refractive index matched transparent hydrogel particles, we are able to resolve the fluid motion even in the vicinity of the particles. The rotation and motion of the hydrogel particle itself is retrieved from the motion of the tracers adhering to its surface and thus acting as surface markers. The exact particle position is determined in the post-processing and currently includes the knowledge of the particle size and its spherical shape. Tracer trajectories of the fluid flow are transferred to the Vortex-in-Cell-sharp method to reconstruct the Eulerian velocity field. Examples of detecting the particle rotation in a laminar pipe flow and of observing flow instabilities arising in a pulsatile particle-laden pipe flow are presented.

Outlook

The simultaneous measurement of fluid flow and particle motion presents promising results for the better understanding of particle-laden pipe flows. In the presented results, we do not yet utilize the full capabilities of Shake-The-Box processing, with respect to the tracer density concentrations. By working with higher tracer seeding, we will be able to even better resolve the flow field spatially and determine quantities like the wall shear stress. The latter is an important quantity for e.g. the strain of blood-vessel in cardiovascular flows. Another objective is to extend the measurement domain to about $10\,D$ to monitor, for example, the development of the observed turbulent structure in the pulsatile pipe flow.

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