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Bestimmung des Reynoldsspannungstensors mittels MRV – Vergleich mit LDV und Fehleranalyse

Reynolds stress tensor measurements using MRV – Comparison with LDV and sources of error

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Reynoldsspannungstensor, Magnet-Resonanz-Velocimetrie, Peridodic Hill, Turbulenz
Reynolds stress tensor, magnetic resonance velocimetry, periodic hill channel, turbulence

Zusammenfassung

Diese Studie analysiert die Zuverlässigkeit und die systematischen Fehler der Magnetresonanz-Velocimetrie (MRV) mit ICOSA6 (sechs ikosaedrische Kodierrichtungen) bei der Quantifizierung des Reynolds-Spannungstensors (RST) komplexer turbulenter Strömungen. Die Methode basiert auf dem turbulenzbedingten Signalverlust, der in Abhängigkeit zur Stärke der Strömungsfluktuation und der Geschwindigkeitsempfindlichkeit der MRV-Messung steht. Die RST-Werte werden daraus mittels voxelweisen Fittings der Signalintensitätswerte an eine dreidimensionale Gauß-Verteilung berechnet.
Um die systematischen Fehler dieser MRV-Methode zu identifizieren, wurden RST-Messungen in einem periodischen Hügelkanal bei einer Reynoldszahl von Re = 29.500 durchgeführt. In einer 2D-Schicht mit 1 mm Auflösung in der Ebene wurden alle sechs RST-Komponenten innerhalb von 8 Stunden gemessen, was 90 000 Datenpunkte ergab. Anschließend wurde eine Laser-Doppler-Velocimetrie (LDV) im gleichen Kanal an ausgewählten Linien innerhalb der gleichen 2D-Schicht durchgeführt, um die MRV-Ergebnisse zu validieren.
Der Vergleich der beiden Datensätze zeigt eine hohe qualitative Übereinstimmung der MRV-Daten mit den Referenzdaten. Es werden jedoch systematische Fehler deutlich. Der bedeutendste Fehlerbeitrag ist der zusätzliche Signalverlust aufgrund höherer Bewegungsordnungen, wie z. B. der Beschleunigung, was zu einer Überschätzung der RST-Werte führt. Darüber hinaus geht der Rekonstruktionsprozess von einer Gaußschen Verteilung

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der Turbulenz aus, was zu einer fehlerhaften Berechnung von nicht-Gaußschen verteilten Turbulenzen führt.
Abgesehen von diesen Schwächen zeigte die Studie, dass die MRV vergleichsweise schnell Turbulenzdaten liefern kann, was besonders in komplexen Systemen ohne optischen Zugang, in denen laseroptische Messungen kaum möglich sind, von Vorteil ist. Der Vortrag auf der Tagung wird einen detaillierten Überblick über die RST-Messungen und den Vergleich mit den Referenzdaten geben. Das Potential dieser Messmethode wird diskutiert und Fehlerquellen werden untersucht. Darüber hinaus werden Ansätze zur Überwindung der identifizierten systematischen Fehler und aktuelle Fortschritte bei der RST-Quantifizierung mittels MRV vorgestellt. Der Konferenzbeitrag basiert auf Ergebnissen, die kürzlich in einem Fachjournal veröffentlich wurden (Schmidt et al., 2021).

Abstract

This study analyzes the reliability and systematic errors of magnetic resonance velocimetry (MRV) with ICOSA6 (six icosahedral encoding directions), which is a promising approach to rapidly quantify the Reynolds stress tensor (RST) of complex turbulent flows. The method is based on turbulence-induced signal loss, which is connected to the strength of flow fluctuation and velocity sensitivity of the MRV measurement. RST values are then calculated by the voxel-wise fitting of the signal intensity to a three-dimensional Gaussian distribution.
RST measurements are performed in a periodic hill channel at Reynolds number of Re = 29.500. In a 2D slice with 1mm in-plane resolution, all six RST components were measured within 8 hours, resulting in 90 000 data points. Subsequently, laser doppler velocimetry (LDV) was performed in the same channel at selected lines within the same 2D slice to validate the MRV results.
Comparing both data sets indicates a high qualitative agreement of MRV data with the reference data. However, systematic errors become apparent. The most significant error contribution is the additional signal loss due to higher orders of motion, such as acceleration, resulting in an overestimation of RST values. Furthermore, the reconstruction process assumes a Gaussian distribution of turbulence resulting in an erroneous calculation of non-Gaussian distributed turbulence.
Besides these weaknesses, this study reinforces that RST MRV can rapidly provide turbulence data from complex flows, which is especially advantageous in opaque systems where laser-optical measurements are hardly possible. The presentation at the meeting will give a detailed overview of the RST measurements and the comparison with the reference data. The potential of this measurement method will be discussed, and sources of errors are examined. Furthermore, approaches to overcome the identified systematic errors and recent progress in RST quantification using MRV will be presented. This conference contribution is based on the work and results from a recently published paper (Schmidt et al., 2021).

1. Introduction

Qualitative and quantitative estimation of Reynolds Stress Tensor (RST) is desirable for the investigation of turbulent flows. Common methods serving as ground truth for RST quantification are hot wire anemometry (HAW) and Laser Doppler Velocimetry (LDV), both being one-dimensional (1D) measurement methods (Shirai et al., 2006, Samie et al., 2018). However, physical or optical access to the flow field is required, and both methods result in low data rates due to coincidence filtering. Other optical methods such as Particle Imaging Velocimetry (PIV) or Particle Tracking Velocimetry (PTV) can provide two- or three-dimensional turbulence data but suffer from high random errors (Wilson and Smith, 2013). Besides, experimental setups are complex and require seeding and enormous resources for data (post) processing, especially measurements of more than two dimensions and three RST components.

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Magnetic Imaging Velocimetry (MRV) is a promising method to overcome some of these shortcomings of the established methods. It does not require optical or physical access to the flow field, as well as no seeding. MRV can directly encode the RST components in its signal, making coincidence filtering unnecessary and time for data post-processing comparatively short. However, several sources of error are specific to MRV, and many of these contributions might not have been known yet. Therefore, this study aims to analyze errors systematically in MRV-RST quantification.

MRV utilizes magnetic field gradients to enforce a velocity-dependent phase shift of the MRV signal. The strength of the dephasing results from the linear relationship between the first moment of the magnetic field gradients (m_1) and the flow velocity. Furthermore, in turbulent flows, this dephasing results in signal attenuation that allows quantifying the RST. Using an encoding scheme with six icosahedral encoding directions (ICOSA6), the components of the RST can be calculated from six measurements with the encoded first moment m_1^{enc} and one reference measurement with m_1^{enc} = 0 (Zwart and Pipe, 2013; Haraldsson et al., 2018):

$$m_1^{enc}\begin{bmatrix} 0 & 0 & 0 \\ 1/sqrt(1+\psi^2) & 0 & \psi/sqrt(1+\psi^2) \\ 1/sqrt(1+\psi^2) & 0 & -\psi/sqrt(1+\psi^2) \\ \psi/sqrt(1+\psi^2) & 1/sqrt(1+\psi^2) & 0 \\ -\psi/sqrt(1+\psi^2) & 1/sqrt(1+\psi^2) & 0 \\ 0 & \psi/sqrt(1+\psi^2) & 1/sqrt(1+\psi^2) \\ 0 & -\psi/sqrt(1+\psi^2) & 1/sqrt(1+\psi^2) \end{bmatrix} \text{with } \psi = (1+sqrt(5))/2$$

The calculation of RST using this method is based on three assumptions: a Gaussian velocity distribution within the single voxels, homogeneous flow conditions within the single voxels, and a sufficiently large Lagrangian integral timescale. The signal magnitude of a single voxel can then be expressed as:

$$\left|S(\vec{k}_v)\right| = |S_0| \cdot \exp\left(-\frac{1}{2}\,\vec{k}_v\,\Sigma^{-1}\,\vec{k}_v\right)$$

where $\vec{\mathrm{k}}_\mathrm{v} = \gamma\left[\mathrm{m}_{1,\mathrm{x}}\ \mathrm{m}_{1,\mathrm{y}}\ \mathrm{m}_{1,}\right]^\mathrm{T}$ is the three-dimensional encoding vector, γ is the gyromagnetic ratio, Σ the variance of the 3-dimensional Gaussian, and S_0 the signal of a velocity-compensated measurement.

However, the dynamic range of such measurement is limited: If the m_1^{enc} is strong enough to display signal attenuation in low-turbulent areas, the signal will fall under the noise level in high-turbulent regions and vice versa. Therefore, repeating the measurement with different m_1^{enc} values can increase the dynamic range, which Elkins et al. (2009) demonstrated in a component-wise RST measurement. In this approach, each RST component was measured and reconstructed separately. In contrast, the ICOSA6 encoding scheme requires a three-dimensional reconstruction of the RST values, including all six measurements for calculating the single components, which results in increased precision of the measurement.

Therefore, RST measurements presented in this study are using the ICOSA6 encoding with several m_1^{enc} values measured. In this way, the best possible measurement uncertainty and dynamic range are reached, which allows for a detailed analysis of other systematic errors contributing to the measurement.

2. Experimental setup and methods

A periodic hill channel is selected as a test case to demonstrate the capabilities of the described method for turbulence quantification. Measurements were performed at a hill Reynolds number of 29,500. Furthermore, LDV data collected at the same setup serves as a ground truth. The setup is displayed in Fig. 1.

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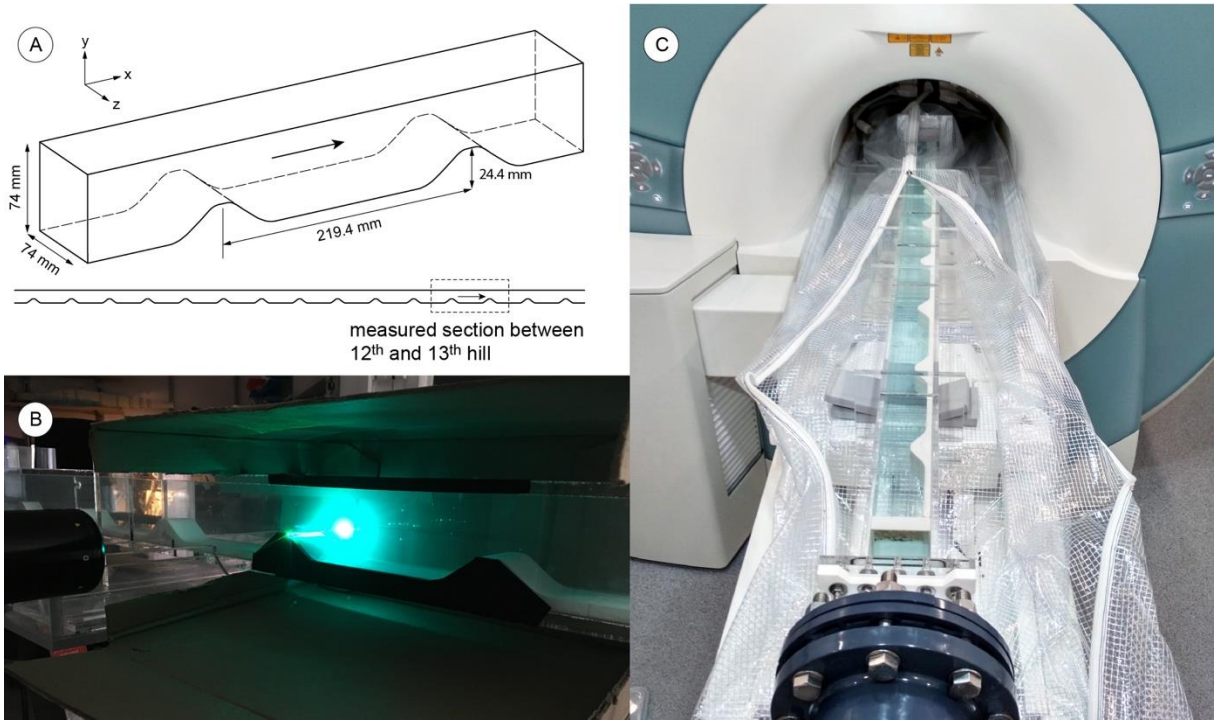

Fig. 1**:** Experimental setup, A: Schematic of the periodic hills, B: LDV measurement, C: Channel inside the MRI scanner.

2.1 Periodic Hill channel

The geometry of the hills is similar to the Ercoftac case 81 “Flow over periodic hills”, described in Temmerman & Leschziner (2001) and Jang et al. (2002), which initially describes a 2D problem, namely for CFD applications. If used for experimental investigations, the channel must have enormous dimensions to avoid 3D effects, which would not fit inside the MRI scanner. Consciously accepting the 3D effects, the test case was scaled to a square cross-section of 74 mm. This results in a maximum hill height of 24.4 mm and a span width of 219.4 mm. With a channel length of 3 m, 15 identical consecutive hills were placed in the channel. Measurements were performed between the 12th and 13th hill to provide sufficient flow periodicity.

Two 5.5 kW centrifugal pumps connected to a 1000 l water tank provide the required flow rate. The working fluid is purified water containing 1 g/l Copper sulfate for the MRV and Vestosint with 5µm particle size for the LDV experiments. The flow rate was controlled using an ultrasound flow rate sensor (Deltawave C-F, Systec Controls, Puchheim, Germany) with a total tolerance of 1.8 l/min or 1.5 % of the measured flow rate. Furthermore, temperature sensors and pressure transducers installed in the flow system ensure monitoring of the flow conditions. The experiments run at 255 l/min flow rate and 22°C, resulting in a hill Reynolds number of 29,500.

2.2 MRV measurement

Measurements were performed on a 3T MRI system (Magnetom Tim Trio, Siemens, Germany) with 40 mT/m maximum gradient amplitude and 200 T/m/s maximum gradient slew rate. A phase-contrast Gradient recalled Echo (GRE) sequence with optimized gradient design (Nishimura et al., 1991) and ICOSA6 encoding scheme was averaged 256 times and repeated with 12 m_1^{enc} values to calculate the RST values. Furthermore, measurements without flow but otherwise identical settings were performed to correct background phase errors. For further details on the sequence, the reader is referred to Schmidt et al. (2020).

Using this method, a 6 mm thick 2D slice reaching from the beginning of the 12th hill to the end of the 13th hill in the axial direction and capturing the whole channel height at 1 mm in-plane resolution was captured in less than 8 hours, including the flow off measurements.
Two receive-only radiofrequency coils were placed below and above the measured section to receive the MRV signal. Their data was combined using sensitivity maps obtained by the ESPIRiT approach (Uecker et al., 2014). RST components were then quantified from a three-dimensional Gaussian fit of all 12 $\mathrm{m}_1^{\mathrm{enc}}$ encoding data after being normalized with the reference measurement. If the normalized signal magnitude was less than 10%, data points were excluded from the fit to avoid noise significantly distorting the results.

2.3 LDV measurements

A FiberFlow system (Dantec dynamics, Skovlunde, Denmark) with a 2-velocity component configuration in coincidence mode and 250 mm optics was used at the same setup and FOV to collect LDV data as ground truth to compare with the MRV data. Six representative lines along the channel height were measured with 0.5 mm step size. At each position, 15,000 to 85,000 valid coincidence samples were collected within 100 s dwell time. For one line, the FOV was moved ± 3 mm along the channel width to examine flow variations inside the MRV measurement volume, as it is significantly wider than in the LDV measurements. However, variations were not measurable, so only the center position was measured in the remaining lines.

Two velocities components ($\bar{\mathrm{u}}_\mathrm{x}$, $\bar{\mathrm{u}}_\mathrm{y}$), three RST components ($\overline{\mathrm{u'_x u'_x}}$, $\overline{\mathrm{u'_y u'_y}}$, $\overline{\mathrm{u'_x u'_y}}$), and two higher statistical moments (skew and kurtosis) were obtained. Bias between fast and slow-moving samples was removed by weighting each velocity sample with their transit time. The data was processed using the BSA Flow Software (Dantec dynamics, Skovlunde, Denmark) and Matlab 2018a (The Mathworks, Natick, USA).

3. Results

Figure 2 displays the 2D3C velocity and 2D6C RST data measured with MRV. This data (approx. 90,000 data points) was sampled in less than 8 hours. In comparison, capturing the six lines using LDV (approx. 1,400 data points) took over 15 h. This illustrates the capability of MRI to capture data very quickly.

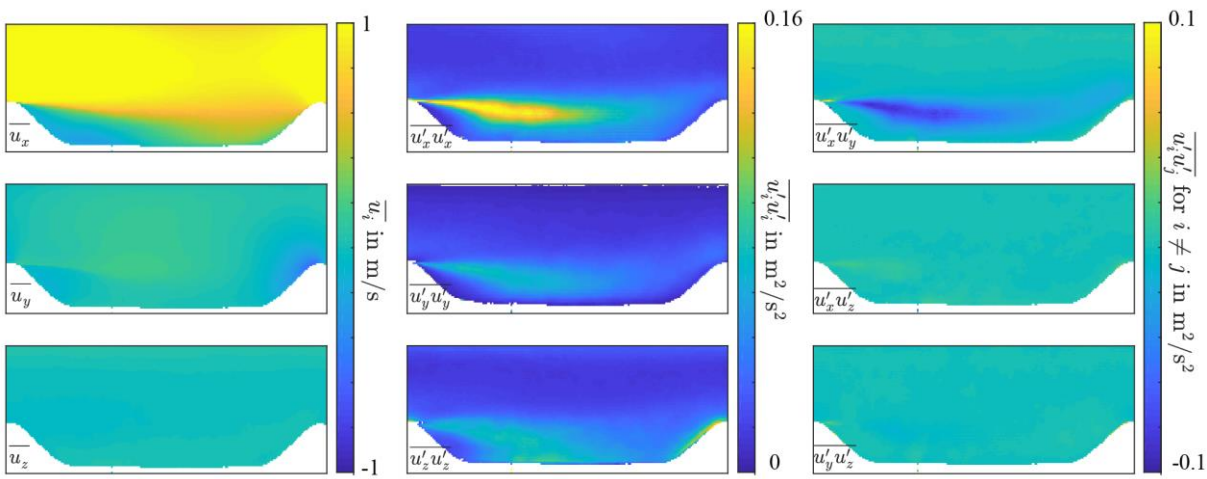

Fig. 2**:** Results of 2D3C velocity and 2D6C RST data measured using MRV.

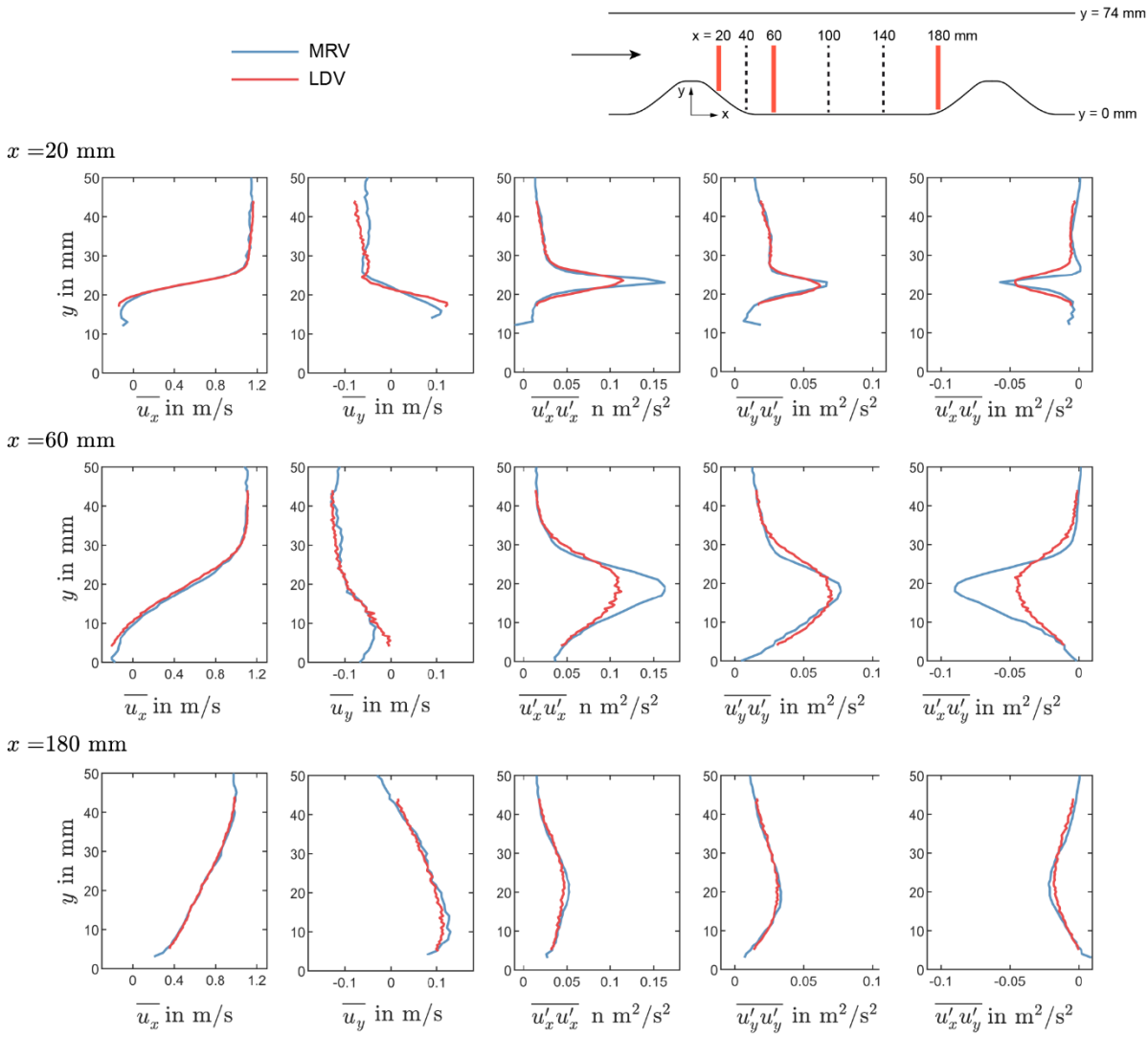

Fig. 3: Comparison of MRV and LDA velocity and RST data at selected lines. The scheme at the top-right displays position of the lines measured using LDA. Dashed lines are not displayed in this figure.

In Fig. 3, the MRV data are plotted with the available LDV data (six lines, two velocity components, and three RST components) to identify the systematic errors of the MRV method. The velocity data agree well, except for some areas above the crest of the hills. However, these deviations are within a 3% range of the axial bulk velocity at the crest of the hills. Stronger deviations arise in the RST data. The $\overline{u'_x u'_x}$ values are up to 0.06 m²/s² higher than in the LDV data, especially in the shear layer downstream of the hill. This effect is also observable in $\overline{u'_x u'_y}$, while the $\overline{u'_y u'_y}$ component shows a higher quantitative agreement.

4. Discussion

MRV and LDV data show good qualitative agreement; however, significant quantitative variations occur in some regions. It is assumed that these deviations arise from systematic errors of the MRV measurements, which will be discussed briefly in the following:

- Spatial filtering caused by the finite voxel size: RST measurements are performed assuming that the velocity distribution inside the voxels is homogenous. However, in regions with strong velocity gradients, additional intra-voxel phase dispersion may occur due to inhomogeneous velocity distribution, resulting in an additional signal loss. In these measurements, it is assumed that this error is most significant at the crest of the hill, where the shear layer is smaller than the voxel size.

- Bias caused by higher orders of motion: The MRV method used in this study (GRE) uses a so-called frequency encoding for shortening the measurement time significantly. However, such methods are sensitive to higher orders of motion along the direction of the frequency encoding. Similar to the dephasing due to velocity variations, higher orders of motion such as acceleration also contribute to the signal loss. In this measurement, the frequency encoding was performed along the x-direction, which corresponds with the observed higher deviations in $\overline{\mathrm{u'_x u'_x}}$ and $\overline{\mathrm{u'_x u'}_y}$.
- Bias caused by non-Gaussian turbulence: As mentioned in section 1, MRV RST assumes a Gaussian distribution of the random velocity fluctuations. Fig. 4 shows the effect if the distribution in the test case would have been indeed Gaussian distributed. The LDV data is therefore fitted on a Gaussian distribution instead of directly calculating the RST values. As a result, the deviation in the $\overline{\mathrm{u'_y u'_y}}$ component almost disappears. In contrast, they are only slightly reduced in the other components, where errors arising from higher orders of motion are assumed to be the most dominant source of error.

For a more detailed discussion of these and other errors, the reader is referred to the corresponding paper of Schmidt et al. (2021). The latter two errors are assumed to have the highest impact on the quality of the MRV RST measurement. Signal loss due to higher orders of motion can be reduced by adjusting the method's parameter or using a single-point-imaging (SPI) encoding instead of a GRE, which has a much lower sensitivity to these errors but may suffer from other drawbacks. Errors caused by non-Gaussian distribution are much harder to eliminate as it is a central assumption of the RST MRV method. However, with ICOSA6 encoding and several m_1^{enc} encodings measured, it may be possible to estimate the quality of the fit and, thereby, determining how strong the impact of this assumption is.

5. Conclusion

MRV RST measurements were performed in a periodic hill channel at a hill Reynolds number of 29,500. A combination of ICOSA6 encoding and several repetitions of the measurement with different m_1^{enc} values resulted in a higher dynamic range and lower measurement uncertainty. In comparison with LDV measurements, the MRV data showed excellent qualitative agreement. However, several sources of errors were identified, leading to quantitative deviations between both measurements.

Higher orders of motion and the assumption of a Gaussian distribution of the velocity were identified as the most prominent sources of errors. While the first can be reduced by adjusting the measurement parameter, the latter is much harder to eliminate. However, it is assumed that the method used in this paper can at least provide information on how well the Gaussian assumption is applicable.

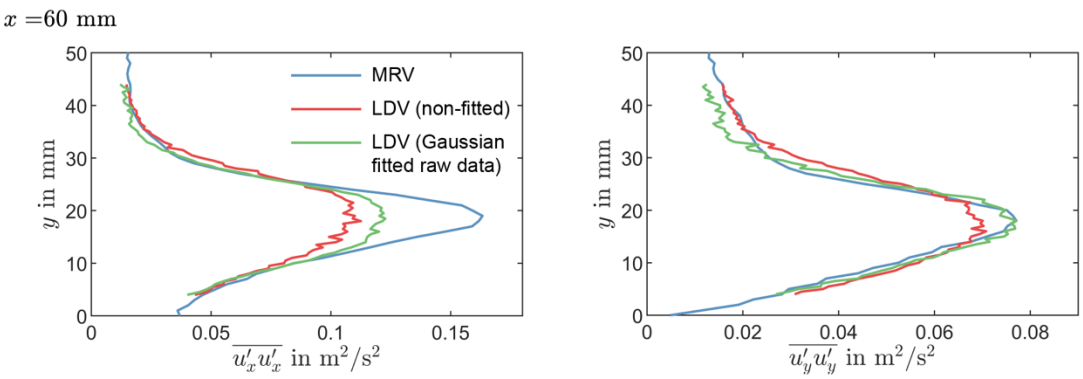

Fig. 4**:** Influence of assuming a Gaussian velocity distribution in the reconstruction of MRV RST.

In comparison to LDV, the proposed MRV method allows comparatively rapid measurements of complex turbulent flows. A complete 2D slice with approx. 15,000 data points were sampled

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in less than 8 hours, and results were qualitatively in good agreement with the LDV measurements. However, several sources of errors were identified, and further work is needed to improve MRV RST quantification.

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