Incipient motion on regular substrates: Impact of lattice structure

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Abstract

We consider a single mobile bead subjected to a shear flow. The bead rests on fixed spheres arranged on triangular lattices. The particle Reynolds number is kept much smaller than 1. To study the impact of the geometry on incipient motion, we vary the spacing between the spheres in the substrates and calculate the critical Shields number using numerical simulations. We find that for loosely packed substrates the effect of the substrate structure is significant, while for densely packed substrates the influence is small. To compare the result with existing model (Agudo et al., 2017: "Shear-induced incipient motion of a single sphere on uniform substrates at low particle Reynolds numbers", J. Fluid Mech. 825:284-314) we also calculate the effective zero level of the fluid flow. We find a good agreement between the model and the numerical simulations.

Introduction

Many environmental phenomena for example erosion, sediment transport and dune formation (Herrmann 2007, Wierschem et al. 2008, Groh et al. 2008, Carneiro et al. 2011) as well as technical applications such as positioning and assembling of particles (Yin et al. 2001, Bleil et al. 2006, Nguyen and Yoon 2009), and pneumatic conveying (Stevanovic et al. 2014) depend on the onset of particle motion. Due to the relevance in a large number of areas, onset of particle movement has been studied intensively, focusing mainly on turbulent flows, since they occur frequently in natural systems (Shields 1936, Chang 1939, Yalin and Karahan 1979, Ali and Dey 2018). In order to gain a more detailed understanding, the focus has recently shifted to the study of incipient motion in laminar flow conditions (Charru et al. 2004, Loiseleux 2005, Ouriemi 2009, Derksen 2011, Agudo and Wierschem 2012, Seizilles 2013, Agudo et al. 2014, Agudo et al. 2017, Deskos and Diplas 2018, Agudo et al. 2018, Topic et al. 2019).

Very few studies have focused on the incipient motion where the substrate on which the particle moves has a regular structure (Agudo and Wierschem 2012, Agudo et al. 2014, Agudo et al. 2017, Agudo et al. 2018, Topic et al. 2019). Unlike the motion on disordered substrates, where averages or probability distributions are considered, regular substrates allow a deterministic prediction of the incipient motion, simplifying the study of the effect of lattice structure. Incipient motion under laminar flow conditions on substrates composed of fixed spheres arranged on a triangular lattice (spheres in contact) and quadratic lattices (varying gap between

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spheres) has been studied experimentally by Agudo and Wierschem 2012. Systems with quadratic arrangements of spheres were numerically simulated and a model with a good agreement with the simulations was developed (Agudo et al. 2017, Topic et al. 2019).

Here, we numerically simulate a system with the substrate spheres arranged on a triangular lattice with spacing between the spheres. We calculate the critical Shields number and compare it with the results on a quadratic lattice and the prediction of the model developed by Agudo et al. 2017. In order to make this comparison we also calculate the amount of penetration of the fluid below the top of the substrate spheres.

Description of the problem

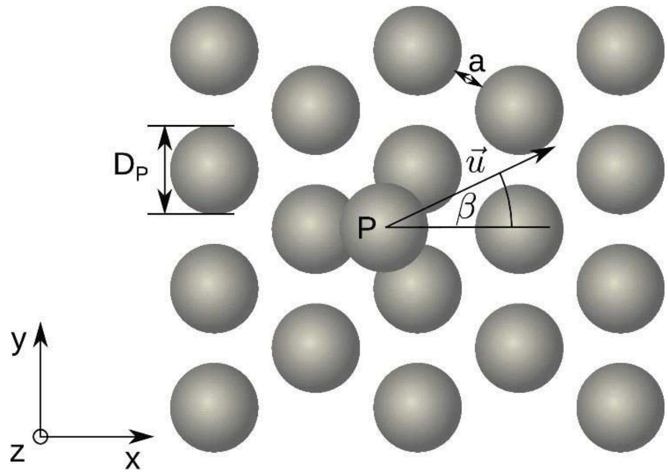

Fig. 1: Arrangement of spheres in the system and the names of variables. The velocity of the top plate, $\vec{u}$, which induces a shear flow in the system is also shown. The z axis points towards the reader.

The arrangement is shown in Fig. 1. The substrate spheres and the mobile bead have equal diameters, D_P. The substrate spheres are arranged on a triangular lattice, with the variable spacing a between neighbors. The substrate particles are fixed on a flat surface. The mobile bead rests on three supporting particles. Above this assembly is again a flat horizontal plate that moves with velocity $\vec{u}$, whose orientation is described by an angle β (see Fig. 1.). The height between the top of the substrate spheres and the moving plate is h. In between the plates is a fluid, and the top plate induces a shear flow. The gravitational acceleration, g, acts opposite to the z direction.

To characterize the role of inertia on the incipient motion we use the particle Reynolds number which for our system can be defined as

$$Re_P = \frac{\dot{\gamma} D_P^2}{\upsilon} \qquad (1)$$

where $\dot{\gamma} = |\vec{u}|/h$ is the shear rate of the fluid, and υ is the kinematic viscosity. The shear rate is chosen such that particle Reynolds number is 0.01. For particle Reynolds numbers smaller than about one, it was shown experimentally that the condition for incipient motion is independent on the Reynolds number (Agudo and Wierschem 2012).

We characterize the condition for the incipient motion with the Shields number

$$\theta = \frac{\tau}{(\rho_s - \rho_f) g D_P} \qquad (2)$$

where τ is the shear stress, ρ_s is the density of the mobile bead and ρ_f is the density of the fluid. For our case $\tau = \rho_f \upsilon\dot{\gamma}$, and therefore

$$\theta = \frac{\upsilon\dot{\gamma}}{(\rho_s/\rho_f - 1)gD_P} \tag{3}$$

Simulation and data analysis

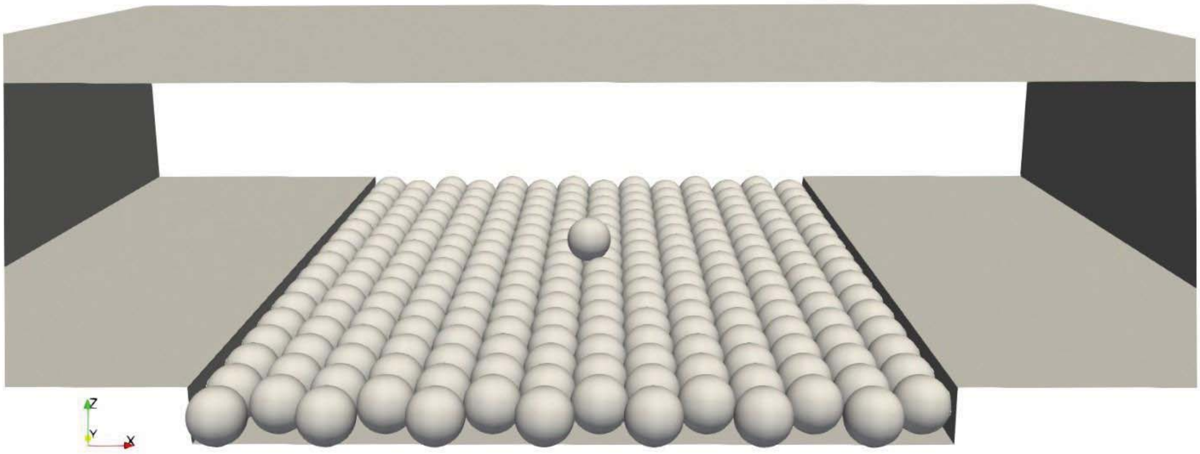

Fig. 2: Simulation domain and particle arrangement in numerical simulation for $a/D_P = 0.0493$, for the measurement of the critical Shields number. The half spheres outside of the domain are periodic images.

The simulated geometry is shown in Fig. 2. The steps left and right of the substrate are added in order to have two-dimensional inflow and outflow conditions. The fluid moves from the left to the right. The vertical surfaces on the leftmost and rightmost part of the domain are the inlet and outlet, respectively. The periodic boundary conditions are imposed on the front and back boundaries of the domain. On the remaining surfaces a no slip boundary condition is observed. The simulation and the meshing were performed with OpenFOAM 2.3.1. The mesh is refined near the gaps between the spheres and the contacts. For further details see in (Agudo et al. 2017).

From the simulations, the values of the force and the torque due to the fluid motion acting on the particle are calculated. The torque is measured with respect to the axis of rotation passing through the contact points of the mobile bead with the two substrate spheres in the downstream direction. As we assume that particle starts its motion by rolling (Agudo et al. 2017), the torque balance for the incipient motion is calculated with respect to the axis passing through the contact points. The torque balance includes the buoyancy, weight of the mobile bead and the torques due to fluid motion. From a single simulation, we compute the particle density which satisfies the torque balance. This particle density is then used in equation (3), in order to compute the critical Shields number.

We define the effective zero level, z_0, of the fluid velocity as the height at which the extrapolated linear velocity profile, which develops above the substrate, reaches zero. z_0 is measured relative to the top of the substrate sphere. For positive z_0 the effective zero level is below the top of the substrate spheres. The velocity profile is calculated without the mobile sphere on the substrate. The component of the fluid velocity in the direction of the velocity of the top plate is averaged at many heights above the three central spheres of the substrate that support the mobile bead, producing the velocity profile used in the calculation of the effective zero level.

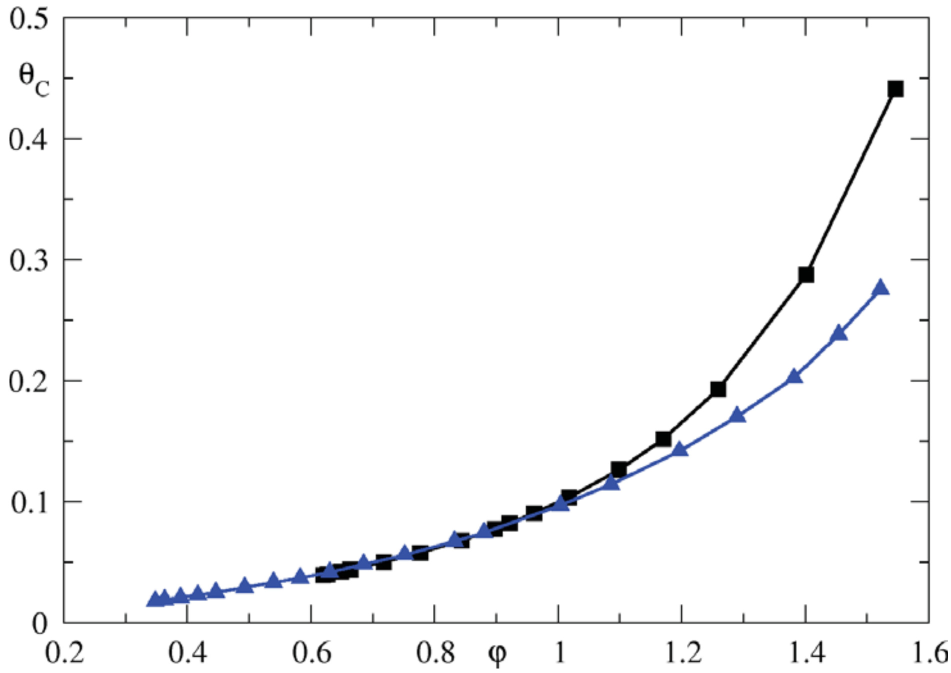

Fig. 3: Dependence of the critical Shields number on the angle of repose on substrates whose spheres are arranged on quadratic (square symbols) and triangular (triangular symbols) lattices. The results for the quadratic lattice are from (Agudo et al. 2017).

Results

To characterize the influence of the structure of the substrate we compare the critical Shields number calculated on a triangular lattice with the values found on the quadratic lattice, at the same angle of repose φ, see Fig. 3. The angle of repose is defined as the angle with respect to positive z direction of the line perpendicular to the axis of rotation and connected to the centre of the mobile bead.

The triangular lattice allows a more compact arrangement of spheres below the mobile bead, therefore the bead may have lower angles of repose compared to quadratic lattice. Similar to the quadratic substrate, the Shields number increases monotonically with the angle of repose. This increase occurs due to two effects: 1) At large angles of repose, larger torques due to fluid are needed for the particle to start moving and 2) The shielding of the mobile bead from the flow by the substrate becomes more pronounced when the bead is closer to the ground. In the range of $0.6 < \varphi < 1$ the critical Shields numbers have nearly the same values on both lattices. For larger angles of repose the deviation steadily increases and reaches 37% relative to the values for the quadratic lattice. Small angles of repose correspond to tightly packed spheres in the substrate, and vice versa. It can be concluded that for tightly packed substrates the influence of the details of the structure of the substrate on the flow properties is small. On the other hand, for looser arrangements, the substrate structure has a significant influence on the condition for incipient motion.

Recently, a model has been developed which predicts the critical Shields number on the quadratic and triangular lattices, (Agudo et al. 2017). This model relies on the knowledge of the effective zero level of the fluid, z_0. Here, we have calculated the effective zero level on a triangular lattice with orientation such that the flow orientation angle is $\beta = \pi/6$, while the velocity of the top plate remains unchanged, see Fig. 4. The velocity profile is fitted in the range between $0.4D_P$ and $4D_P$ above the tops of the substrate spheres.

For the triangularly arranged substrate the effective zero level varies approximately linearly with the substrate gap, similar to the quadratic arrangement. The zero level can be fitted well with the function $z_0/D_P = 0.081 + 0.093\ a/D_P$. For the largest possible gap of the triangular lattice the effective zero level is at slightly lower height (therefore appearing higher in Fig. 4.) than for the quadratic lattice, reaching about 15% of the particle diameter. This effect may be related to the observation in Fig. 3, that at larger gaps and for the same angles of repose the critical Shields number is smaller on triangular lattices than on quadratic. Overall, the effective zero level variations are a small fraction of the particle diameters.

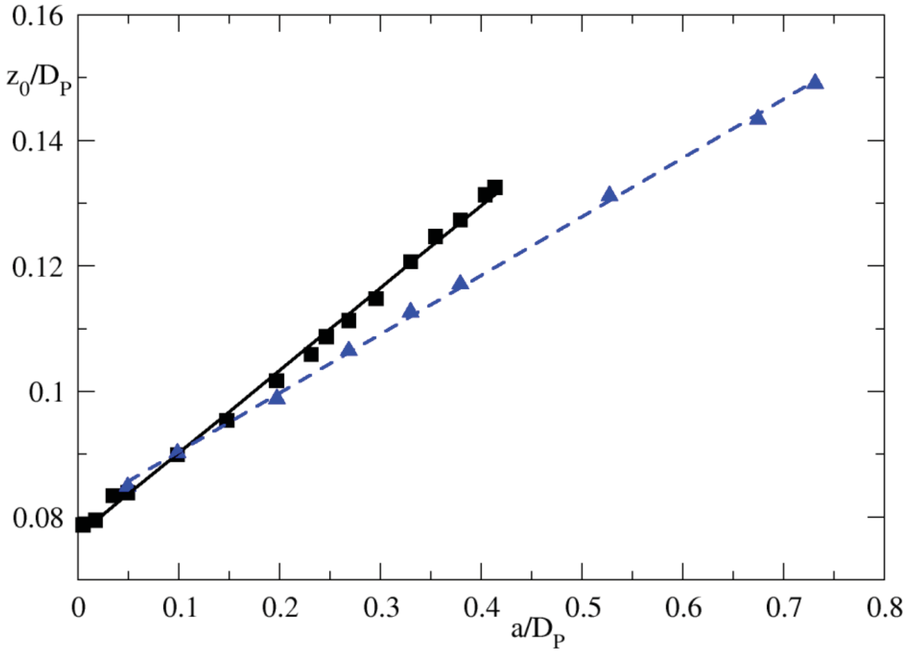

Fig. 4: Dependence of the effective zero level of the fluid on the substrate gap. Squares and triangles correspond to simulations on quadratic and triangular substrates, respectively. The full and dashed lines are linear fits to the data for the quadratic and triangular lattices, respectively. The results for the quadratic lattices are from (Agudo et al. 2017).

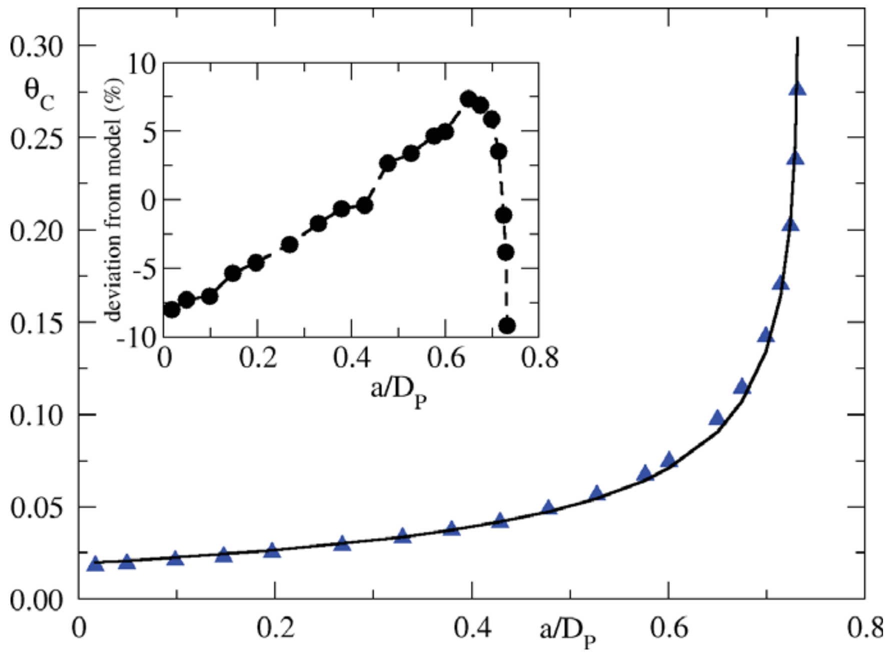

Fig. 5: Comparison between the predictions of the model (Agudo et al. 2017) with the numerical simulations on the triangular lattice. In the main plot, the full line is the prediction of the model and triangles are the results of the simulation.

With the measured effective zero level it is now possible to compare the predictions of the model (Agudo et al. 2017) with the Shields numbers computed numerically, see Fig. 5. For the smallest gap the model predicts approximately 8% larger values, see the inset of Fig. 5. The deviation then steadily decreases, reaching the minimum at approximately $a/D_P = 0.4$. For larger gaps the difference again increases up to 8%, and the model predicts smaller values than the simulation. For the largest possible gaps, the model again predicts higher values. A similar trend of the deviation of the model with respect to simulations was observed for quadratic substrates, (Agudo et al. 2017). The maximum deviation does not exceed 10%. Therefore, there is a good agreement between the model and simulations.

Conclusions

We have considered the effect of the substrate geometry on the incipient motion of a spherical particle in case of low particle Reynolds number flows. The substrates consisted of fixed spheres arranged on triangular lattices with variable gaps between the spheres.

We have found that for densely packed substrate spheres the critical Shields number on the triangular lattices has values similar to those computed for the quadratically arranged substrates, at the same angle of repose. For looser substrates the critical Shields number is significantly smaller on triangular substrates. We have also measured the effective zero level of the flow, which allowed a comparison of the predictions of the recently developed model with numerical simulations. A good agreement was found between the model and the simulations.

Acknowledgements

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