Fachtagung "Experimentelle Strömungsmechanik"
4. – 6. September 2018, Rostock

Investigations on the possibility of Lattice Boltzmann as a Methodology for Flow Analysis in Subsonic Axial Compressor

Mohammad Mobarak, Mohamed Hussein, Antonio Delgado.
Lehrstuhl für Strömungsmechanik
Friedrich-Alexander-Universität Erlangen-Nürnberg
Cauerstraße 4
91058 Erlangen

Lattice Boltzmann Method, Turbulence Model, Turbomachinery, Subsonic Axial Compressor.

Abstract

Lattice Boltzmann Method (LBM) has recently gained great attention in the flow field modelling, especially in complex medium. One great advantage is the method of handling complex geometries and obstacles in a straight forward particle collision method.

The proposed work is to apply a developed and validated Lattice Boltzmann model for incompressible turbulent fluid flows to a subsonic axial compressor with Reynold's number 300,000. The scheme's stability and its performance at higher Reynold's number were investigated for its validity especially at critical flow regimes in regard to the combined LBM Smagorinsky Large Eddy Simulation (LBMLES) Turbulence model.

The developed LBM model is applied to investigate the flow inside a subsonic axial compressor, innovatively, this is the first time to be achieved in literature using LBM. Worth to mention, the benefit from LBM's particle-obstacle collision treatment was gained to catch the interaction of fluid with moving rotor and fixed stator blades. Achieved without the need of dynamic grid or moving frames with interfaces as in conventional computational methods in turbomachinery. Finally, the flow field was visualized, investigated and the available turbomachinery parameters were studied for different operating conditions. The error with the available experimental data from the study case is also discussed fairly.

Introduction

Lattice Boltzmann Method (LBM) has recently gained much attention in the field of fluid mechanics, including multi-phase, thermal, complex and micro-scale media flows with low Knudsen number, at scales where continuum assumption fails. LBM is a hyper stylized version of the Boltzmann equation explicitly designed to solve fluid-dynamics problems, and beyond, see Succi 2001. The main interest through this work is to investigate the celebrated LBM in the application of turbomachinery, specifically in subsonic axial compressor at Reynolds number 300,000. The technique which LBM deals with moving boundaries is quite interesting, where no moving frames or dynamic grid are required to capture the rotor-stator interaction and the flow physics. The application of LBM in the field of turbomachinery was only found in literature through J. Eggels 1996 and Filipova et al. 2001 - F. Mazzokko et al. 2000. The work from J. Eggels was to investigate the Smagorinsky Large Eddy Simulation (LES) Turbulence model on a baffled stirred tank reactor accordingly with a novel forcing LBM scheme, although the technique and application in this work is much different. The work by Filipova et al. - F. Mazzokko et al. was based on a static rotor blade cascade, where no moving geometries are implied or even a complete rotor-stator stage is analysed. Therefore, the application of a validated LBMLES model in a subsonic axial compressor with the proposed configuration and turbulent flow nature has motivated this work to be a suggested step towards a new alternative technique for complete flow analysis in turbomachinery.

Lattice Boltzmann Method

Lattice Boltzmann Method can be considered as a finite difference scheme of the classical Ludwig Boltzmann's Equation (BE) "equation (1)" - which models the particles interactions through the particle distribution function f - in the discretised form, see Chen, S. and Doolen, G.D., 1998, accordingly with the Bhatnagar-Gross Krook (BGK) approximation for the collision operator Q, see Koelman 1991. Where v is the particle velocity.

$$\frac{\partial f}{\partial t} + v \cdot \nabla f = Q \tag{1}$$

The collision operator Q can then be expressed as follows:

$$Q_i = -\frac{1}{\tau}\left[f_i - f_i^{(equ)} \right] \tag{2}$$

Where, $f_i^{(equ)}$ is the local equilibrium distribution function evaluated from the Maxwell-Boltzmann distribution and τ is the relaxation time for the particles to move from the non-equilibrium to the equilibrium state during collision. The LBE which carries the core procedure of the method can be expressed as:

$$f_i(x + \hat{e}_i \Delta t, t + \Delta t) = f_i(x,t) - \frac{1}{\tau}\left(f_i(x,t) - f_i^{equ}(x,t) \right) \tag{3}$$

The equilibrium distribution function truncated at the second order velocity term evaluated from the Hermite expansion of the Maxwellian distribution function - which satisfies the dynamic collision invariants; number, momentum, particle kinetic energy, see Hussein 2010 - can be expressed as:

$$f_i^{equ} = w_i \rho \left[1 + 3\frac{\hat{e}_i \cdot \hat{u}}{c^2} + \frac{9}{2}\frac{(\hat{e}_i \cdot \hat{u})^2}{c^4} - \frac{3}{2}\frac{\hat{u} \cdot \hat{u}}{c^2} \right] \tag{4}$$

Where, $\hat{e}_i$, w_i, c and u are the discrete unit vectors, the weighting parameter, the lattice speed $c = \frac{\Delta x}{\Delta t}$ and the macroscopic velocity respectively. The lattice can be either D2Q9 or D3Q19 (Figure 1) & (Table 1)

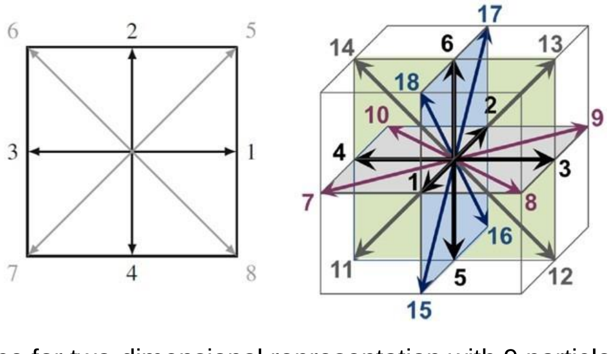


Figure 1: Left: D2Q9 Lattice for two-dimensional representation with 9 particle degrees of freedom, Right: D3Q19 Lattice for three-dimensional representation with 19 particle degrees of freedom (Hussein 2010).

D2Q9		D3Q19	
Direction e_i	Weighting parameter w_i	Direction e_i	Weighting parameter w_i
0	4/9	0	1/3
1, 2, 3, 4	1/9	1, 2, 3, 4	1/18
5, 6, 7, 8, 9	1/36	5, 6, 7, 8, 9	1/36

Table 1: Weighting parameters for D2Q9 and D3Q19 lattices.

The macroscopic density and speed are evaluated from the zeroth and the first moment of the distribution function respectively as:

$$\rho = \sum_i f_i \ ; \hat{u} = \frac{1}{\rho} \sum_i f_i \ \hat{e}_i \tag{5}$$

The Mach number and the lattice speed of sound c_s are expressed as:

$$M = \frac{u}{c_s}; \ c_s = \frac{c}{\sqrt{3}} \tag{6}$$

While the pressure is related to the macroscopic density in the incompressible limit through the isothermal gas relation as:

$$P = \rho c_s \tag{7}$$

Now, the relaxation time is evaluated from the fluid kinematic viscosity v from relation between the Chapman-Enskog expansion of the LBM and the incompressible Navier-Stokes equation, see Succi 2001, which can be written as:

$$\tau = 3v + \frac{1}{2} \tag{8}$$

Sub-grid scale Turbulence Model

Large Eddy Simulation is a computation methodology in which large eddies are computed while small eddies in the sub-grid scale are modelled, see Wilcox 1998. Which is based on the filtering concept of the macroscopic quantities for the Navier-Stokes equations and -on the same basis- of the distribution function ($\overline{f}$) for LBM. The Smagorinsky model - see Smagorinsky 1963 - assumes that the sub-grid scale (residual) stresses $\tau_{ij}{}^R$ follow a gradient-diffusion process - see Wilcox 1998 - through the turbulent viscosity μ_t which is evaluated as follows:

$$\tau_{ij}{}^R = 2\mu_t \overline{S_{ij}}; \ \mu_t = \overline{\rho} v_t = \overline{\rho}(C_s \mathit{\Delta})^2 \sqrt{2\overline{S}_{ij}\overline{S}_{ij}} \tag{9}$$

Where $\overline{S_{ij}}$, $\overline{\rho}$, C_s and $\mathit{\Delta}$ are the filtered strain rate tensor, the filtered density, the Smagorinsky constant and the filter width respectively. According to the work by Hou et al. 1994 the filtered strain rate tensor can be evaluated from the filtered non-equilibrium distribution function through the Non-equilibrium Stress tensor $\overline{\Pi}_{ij}$ as in equation (10). Since the relaxation time is physically and mathematically related to the molecular viscosity, the total relaxation time can include both the molecular and turbulent viscosity. Then solving for the filtered strain rate tensor, the total relaxation time can be written as shown in equation (11).

$$\bar{S}_{ij} = -\frac{3}{2\bar{\rho}\tau_{total}\Delta t}\overline{\Pi}_{ij}\,;\quad \overline{\Pi}_{\mathrm{ij}} = \sum_{\alpha} e_{\alpha i} e_{\alpha j}(\bar{f}_{\alpha} - \bar{f}_{\alpha}^{\,equ}) \tag{10}$$

$$\tau_{total} = 3(\nu_o + \nu_t) + \frac{1}{2} = \frac{1}{2}\left(\sqrt{\tau_o^{\,2} + \frac{18}{\Delta x^2 \bar{\rho}}(C_s \Delta)^2 \sqrt{2\overline{Q}}} + \tau_o\right) \tag{11}$$

Where $\overline{Q}$ is a scalar quantity evaluated from the double contraction of the non-equilibrium stress tensor ($\overline{\Pi}{:}\overline{\Pi}$), and τ_o is the relaxation time calculated from the molecular viscosity.

Model's Stability

To identify the linear stability condition with the implemented turbulence model, the time evolution LBGK Equation with Smagorinsky SGS Turbulence Model can be seen for a time marching filtered node distribution as:

$$\bar{f}_i(t + \Delta t) = \bar{f}_i(t) - \frac{1}{\tau_{total}}\left(\bar{f}_i(t) - \bar{f}_i^{\,equ}(t)\right) \tag{12}$$

This leads to the known stability condition $\tau_{total} > \frac{1}{2}$, and since the positivity of the equipped turbulence mode is always guaranteed, as the backscattering effect is not permitted – the Smagorinsky constant and $\overline{Q}$ are always positive -, the total relaxation time is inherently greater than τ_o.
Although, this stability condition is necessary but not sufficient, as the scheme stability is affected by the compressibility limitations regarding the lattice macroscopic speed $u < 0.3c_s$ and the velocity gradients.

Study Case: Subsonic Axial Compressor

As for the case study, the LBMLES model is applied on the ZVVC-Czetch highly loaded axial flow compressor - see Cyrus 1998 -. The compressor design was based on a uniform spanwise work distribution. The case of interest was the one named as Case R in the work by Cyrus 1998, where no inlet guide vanes are implemented, also the stator vanes are fixed, although the rotor blades are variable in this case (Figure 2 - Left). For the current work, the rotor setting with angle $\Delta\gamma_R = 0$ are used but for different flow coefficients, scanning a certain speed line (1500RPM) on the compressor map (Figure 2 - Right). The compressor's data can be found in detail in the publication by Cyrus 1998, also the test rig illustration can be seen in the work by Cyrus 1996.

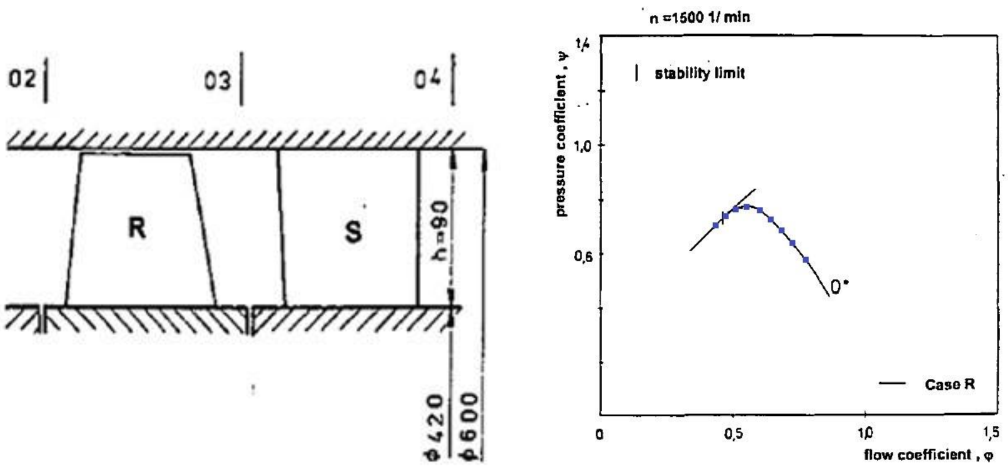


Figure 2: Left: Single stage Axial Compressor Rotor-Stator (Case R), after Cyrus 1998. Right: Speed line for the Off-design case just showing the flow coefficient of the sampled points for LBMLES simulations, the speed line is for the experimental R-case with 0° rotor's setting angle, after Cyrus 1998.

The obstacle geometry of the Rotor-Stator cascade for the LBMLES simulations and the computaional domain are shown on Figure 3.

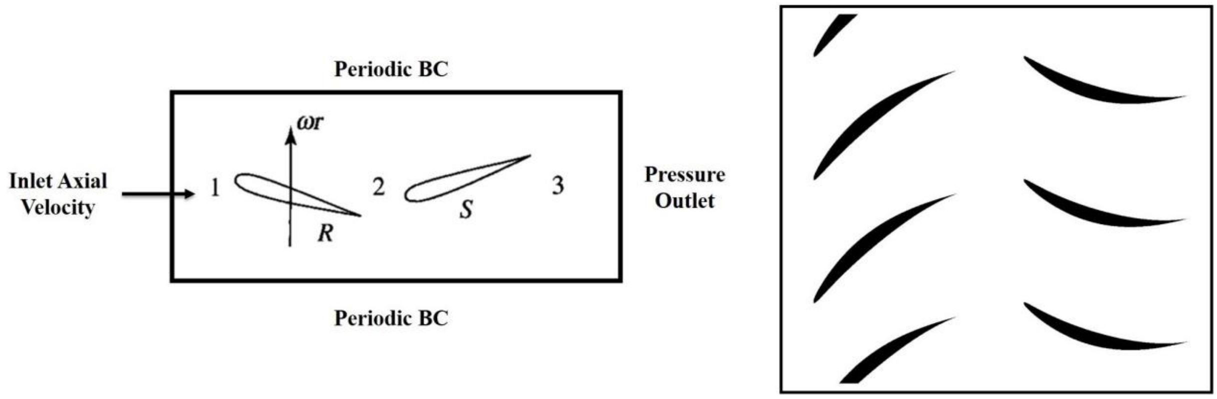


Figure 3: Left: Computational domain of the Rotor stator case. Right: Obstacle Geometry of the Rotor Stator for the LBMLES.

The flow field can be visualised through the instantaneous velocity magnitudes with stream-lines and the vorticity magnitude of some selected flow coefficients ($\phi = \frac{Q}{AU_t}$, where Q, A and U_t are the volume flow rate, flow annulus area and tip tangential speed) from the study as shown in Table 2. At the surge point Φ=0.47, highly separated flow over the rotor ap-pears, also the flow instabilities with exaggerated perturbations appears past the rotor and across and downstream the stator. At the design point Φ=0.6, less separated and more clean flow are shown. For the flow coefficient Φ=0.72, pressure side separation bubbles appear due to the high negative incidence at the rotor blades.

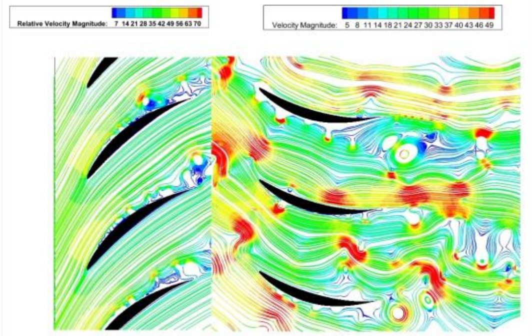
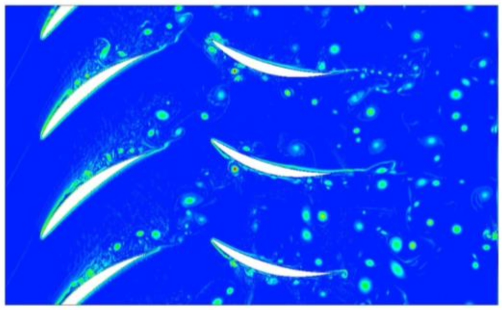
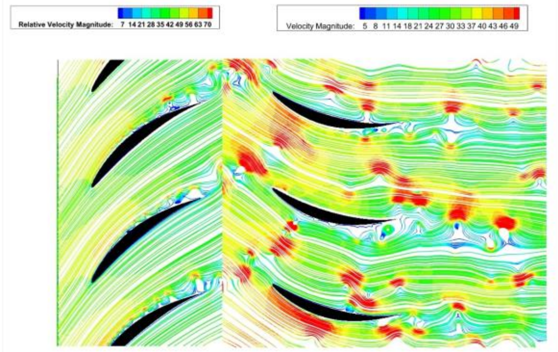
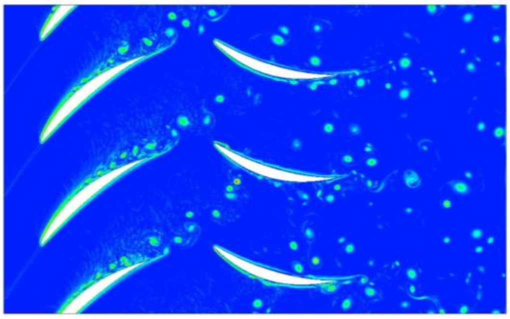
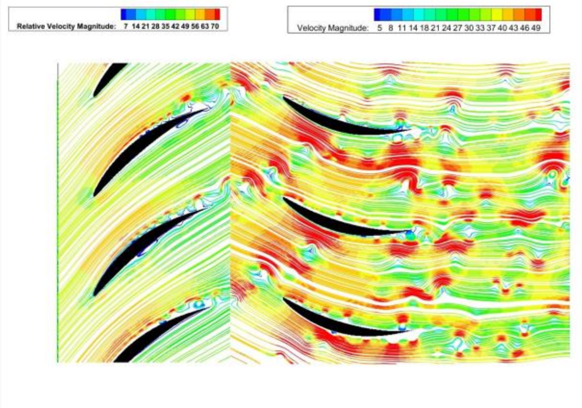
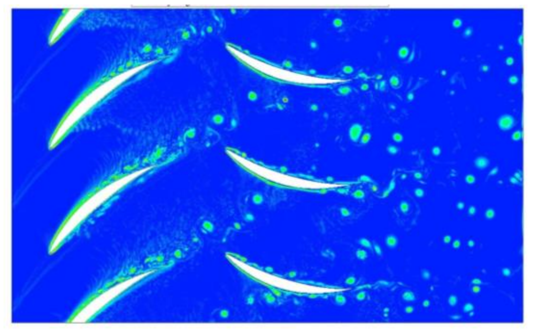
	Velocity Magnitude & Streamlines	Vorticity Magnitude
Φ=0.47		
Φ=0.6		
Φ=0.72		

Table 2: Contours of instantaneous velocity magnitude with streamlines and the vorticity magnitude of some selected flow coefficients.

The compressor stage time averaged absolute velocity and flow angles can be shown on Figure 4 left and right respectively at different operating conditions. At low flow coefficients, higher turning angles appears on the figure, from stage inlet flow angle α_1 to rotor discharge angle α_2 and high velocity magnitude shifting from stage inlet absolute velocity C_1 to rotor discharge absolute velocity C_2. While the inverse appears for higher flow coefficients, where flow turning is less due to the lower difference between the stage inlet absolute velocity C_1 and the rotor's circumferential speed U.

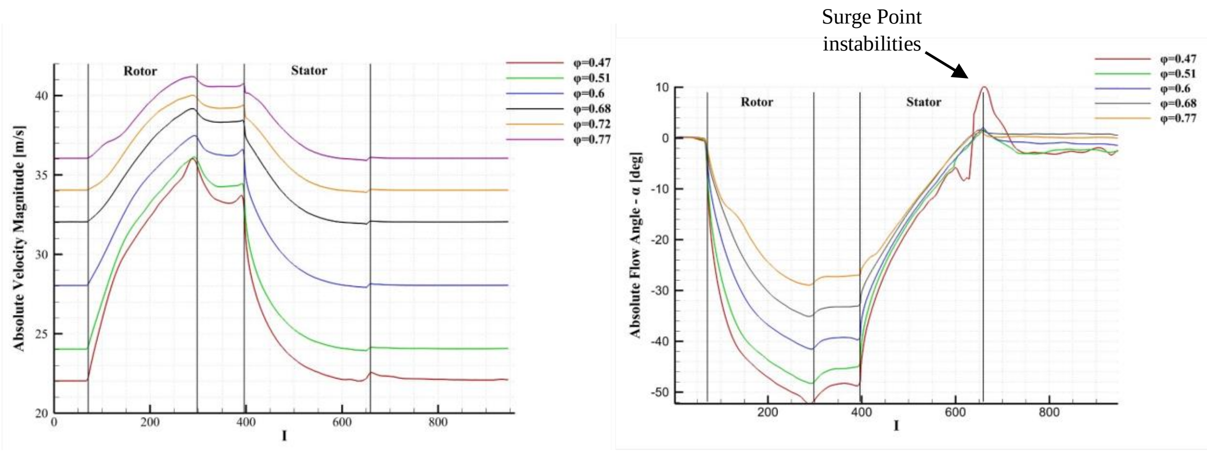


Figure 4: Left: Compressor stage absolute flow velocity magnitude at different flow coefficients. Right: Compressor stage absolute flow angles at different flow coefficients.

While, the rotor's discharge angle - stator's inlet flow angle - α_2 and the stator incidence angles i_{Stator} which is compared with the experimental incidence angles from Cyrus 1998 can be shown on Figure 5. Which appears to have the same trend especially near the design point and even at higher flow coefficients. Quantitively, this deviation from the experimental results can be calculated by the relative root mean square error which gives the value of 8%.

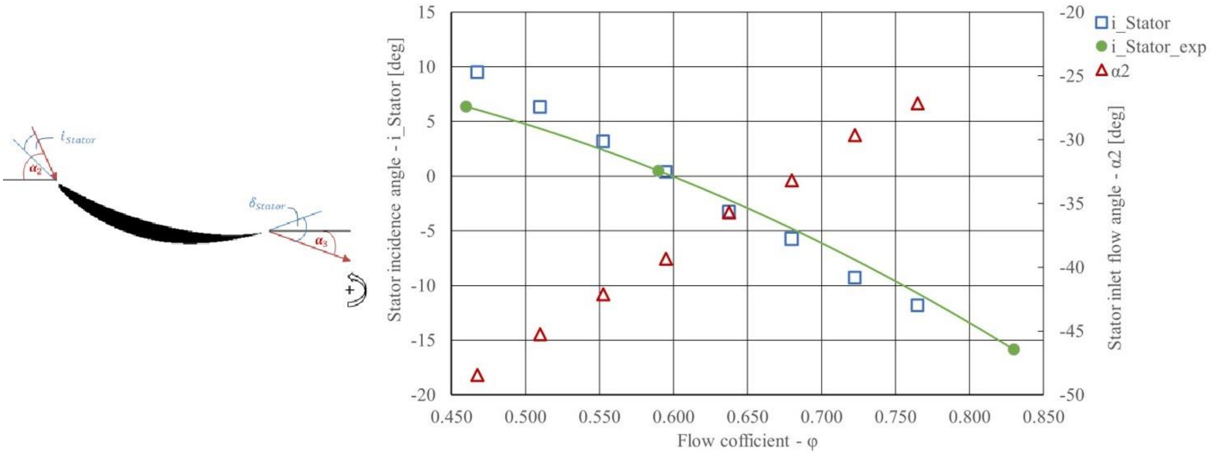


Figure 5: Variation of the stator incidence and inlet flow angles (i_{Stator} and α_2) with flow coefficient φ, based on the simulated operating conditions.

Conclusions

The novel application of the Lattice Boltzmann Method in subsonic axial compressors and turbomachinery in general seems to be much promising, since the way it deals with the moving boundaries and capturing of the Rotor-Stator interaction is way interesting. Although, the field is still some steps from maturity, since it needs some development regarding the multi-node distribution techniques and the implementation of the compressible LBM which are inevitable for this discipline. Simulation results appears to be acceptable with the available experimental data. Further cases in turbomachinery can be studied as a future work with deeper validation experimental and numerical results.

References

Chen, S. and Doolen, G.D., 1998: "Lattice Boltzmann method for fluid flows", Annual review of fluid mechanics, 30(1), pp.329-364.

Cyrus, V., 1996: "Design of axial flow fans with high aerodynamic loading", Forschung im Ingenieurwesen, 62(3), pp.58-64.

Cyrus, V., 1998: "Aerodynamic performance of an axial compressor stage with variable rotor blades and variable inlet guide vanes. ASME Paper, (98-GT), p.151.

Eggels, J.G., 1996: "Direct and large-eddy simulation of turbulent fluid flow using the lattice-Boltzmann scheme", International Journal of Heat and Fluid Flow, 17(3), pp.307-323.

Filippova, O., Succi, S., Mazzocco, F., Arrighetti, C., Bella, G. and Hänel, D., 2001: "Multiscale lattice Boltzmann schemes with turbulence modeling", Journal of Computational Physics, 170(2), pp.812-829.

Hou, S., Sterling, J., Chen, S. and Doolen, G.D., 1994: "A lattice Boltzmann subgrid model for high Reynolds number flows", arXiv preprint comp-gas/9401004.

Hussein M.A., 2010: "On the theoretical and numerical development of Lattice Boltzmann models for biotechnology and its applications", PhD. Technische Universität München, München.

Koelman, J.M.V.A., 1991: "A simple lattice Boltzmann scheme for Navier-Stokes fluid flow", EPL (Europhysics Letters) 15(6): p.603.

Mazzocco, F., Arrighetti, C., Bella, G., Spagnoli, L. and Succi, S., 2000: "Multiscale lattice Boltzmann schemes: A preliminary application to axial turbomachine flow simulations", International Journal of Modern Physics C, 11(02), pp.233-245.

Mobarak M., 2018: "Lattice Boltzmann Method for Flow Analysis in Subsonic Axial Compressor", M.Sc. Thesis - Cairo University.

Succi, S., 2001: "The lattice Boltzmann equation: for fluid dynamics and beyond", Oxford university press.

Wilcox, D.C., 1998: "Turbulence modeling for CFD (Vol. 2, pp. 103-217)", La Canada, CA: DCW industries.